Faculty of Science »
Department of mathematics and statistics

Solution to the Midterm
MAT 2377
Winter 2011

Short Answer Questions

[4] 1.

(a) Since $0 \leq f(x) \leq 1$ for $x \in \{0, 1, 2, 3, 4\}$ and the sum of the probability masses equal to one, thus f is a probability mass function.

(b)

$$E[X] = 0(1/3) + 1(2/9) + 2(2/9) + 3(1/9) + 4(1/9) = 13/9 = 1.4444.$$

(c) $P(X \leq 1) = f(0) + f(1) = 1/3 + 2/9 = 5/9 = 0.5556.$

(d)

$$P(X \geq \mu) = P(X \geq 12/9) = f(2) + f(3) + f(4) = 4/9 = 0.4444.$$

[4] 2.

(a) Let X be the number of detected can among $n = 10$. X has a binomial distribution with $n = 10$ and $p = 0.95$. We want

$$P(X \geq 9) = \binom{10}{9}(0.95)^9(0.05)^1 + \binom{10}{10}(0.95)^{10}(0.05)^0 = 0.9139.$$

We also want its expected value : $E[X] = n\,p = 9.5$.

(b) Let X be number of underfilled cans needed to obtain the first detected can by the monitor. X has a geometric distribution with $p = 0.95$. Its expected value is $E[X] = 1/p = 1.05$.

(c) Let X be number of underfilled cans needed to obtain three detected cans by the monitor. X has a negative binomial distribution with $p = 0.95$ and $r = 3$. We want

$$P(X = 6) = \binom{5}{2}(0.95)^3(0.05)^3 = 0.0011.$$

Multiple Choice Questions :

[1] 1. Let A_i be the event that the i component works. We have $P(A_i) = 0.8$, for $i = 1, 2, 3, 4$. The probability that the system in parallel works is

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 1 - P(A_1' \cap A_2' \cap A_3' \cap A_4') = 1 - (0.2)^4 = 0.9984.$$

[1] 2. Let D be the event that the product is defective. We want

$$\begin{aligned} P(D) &= P(D|M_1)P(M_1) + P(D|M_2)P(M_2) + P(D|M_3)P(M_3) \\ &= (0.02)(0.3) + (0.03)(0.45) + (0.02)(0.25) = 0.0245 \end{aligned}$$

[1] 3. $30 = E[(X-3)^2] = E[X^2 - 6\,X + 9] = E[X^2] - 6\,E[X] + 9 = 30 - 6\,E[X] + 9$, Thus, $E(X) = 9/6$. Thus

$$\sigma = \sqrt{E(X^2) - [E(X)]^2} = \sqrt{30 - (9/6)^2} = 5.27$$

[1] 4. Let C be the event of answering correctly and A be the event that the student knew the answer. We want

$$\begin{aligned} P(A|C) &= \frac{P(A \cap C)}{P(C)} = \frac{P(C|A)P(A)}{P(C|A)P(A) + P(C|A')P(A')} \\ &= \frac{(1)(0.5)}{(1)(0.5) + (0.2)(0.5)} = 5/6. \end{aligned}$$

[1] 5. The probability of choosing 2 defective bulbs when choosing randomly without replacement 3 bulbs from a population of 20 bulbs in which 4 are defective is

$$\frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}} = \frac{8}{95}.$$

[1] 6. Let X be the number of selected individuals that think that the system is adequate. X has a binomial distribution with $n = 10$ and $p = 0.3$. We want

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - \left[\binom{10}{0}(0.3)^0(0.7)^{10} + \binom{10}{1}(0.3)^1(0.7)^9\right] = 0.85 \end{aligned}$$

[1] 7. Let X be the number of calls in 4 weeks. X has a Poisson distribution with a mean of $\lambda = (0.7)(4) = 2.8$ calls. We want

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = 0.0608.$$